



5444, $[0][1]^*$

on a constant negative curvature surface

Chaim Goodman-Strauss and Eugene Sargent

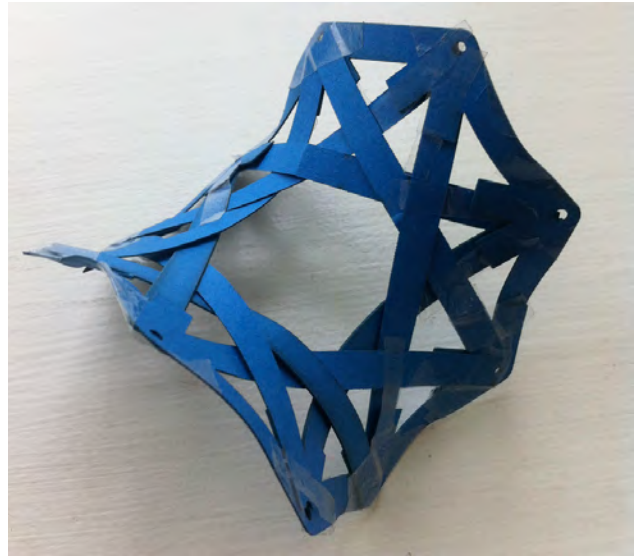
G4GB, March 2014

To Sarah Garvin



The intrinsic geometry of this surface is forced by the lengths and angles of its pieces. Any surface with this intrinsic structure must buckle and bend in this general way. The paper model above is the basis of the specific extrinsic geometry of the final steel sculpture.

We build surfaces of negative curvature, from strips of flat material: The celebrated Gauss-Bonnet Theorem demonstrates that the total curvature of a disk-like region of a surface (for example, one of our units) is precisely captured by measuring the turning excess or deficit around its boundary. For example, consider this decagon with ten 120° angles; as we go around its boundary, we turn 60° ten times, for a total of 600° — an excess of 240° over the customary 360° for flat surfaces. This excess is a precise measurement of the total negative curvature across the decagon.



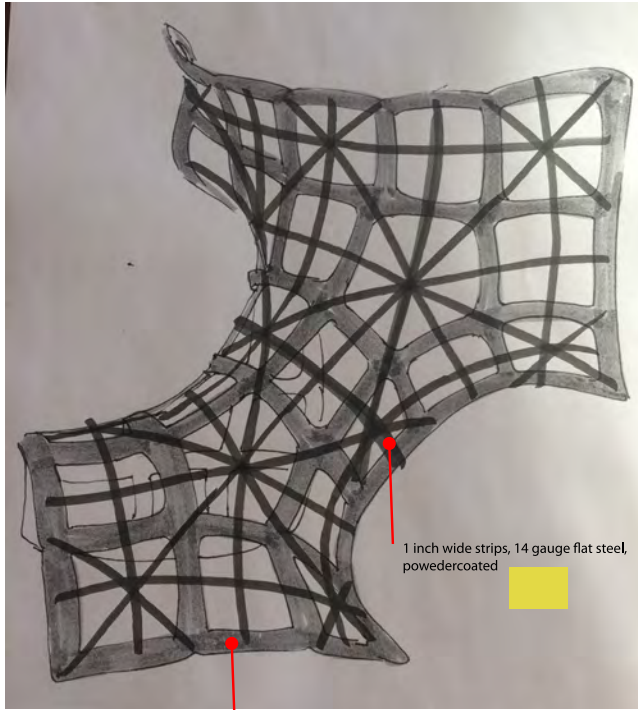
We can control this with exquisite precision. At left is a recent sculpture of a constant negative curvature surface made from pentagons and squares; the angles at the corners are worked out precisely so that the total curvature per unit of area (i.e. the Gaussian curvature) is the same across the entire surface.

The same piece of tiling is shown in the Poincaré disk below, only differing in scale and placement, but fundamentally, intrinsically, the same geometry.



Archimedean of the form
 $*[0][1] (5,4,4,4)$

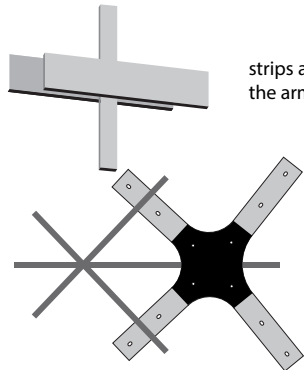
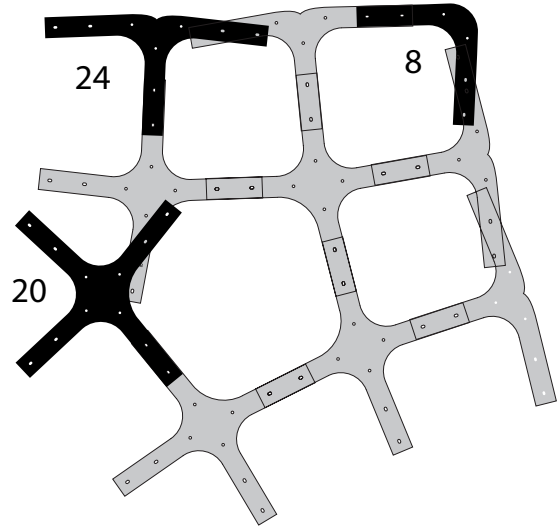
Equilateral polygons;
 the vertex angles are
 85.8676° 102.397°



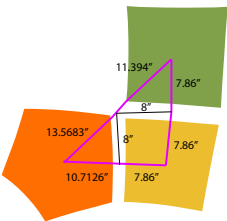
2 inch wide arms, waterjetted from 11 gauge sheet steel, powdercoated

1 inch wide strips, 14 gauge flat steel, powdercoated

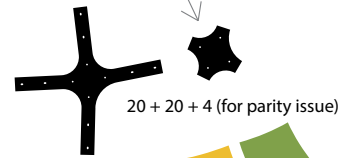
Detail Work ...



strips are hidden in the centers of the arms by overlapping neighboring pieces.

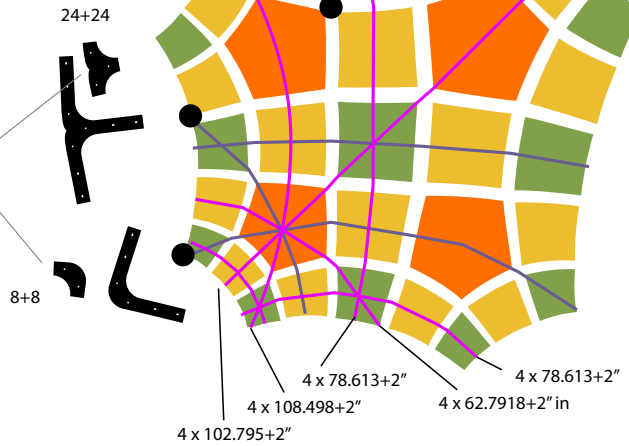


blew these off—were to cover strips at nodes



20 + 20 + 4 (for parity issue)

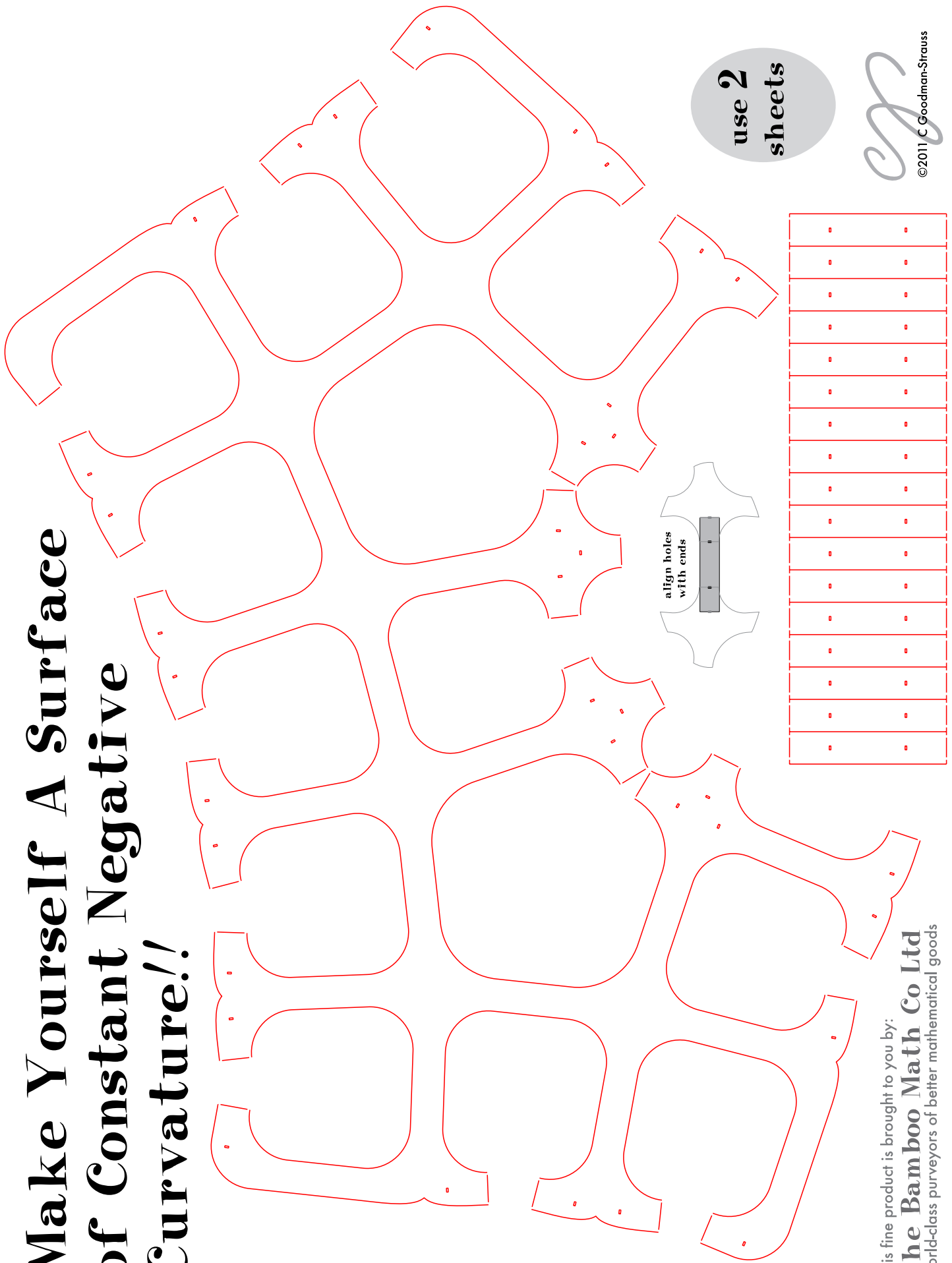
blew these off too



Cut up the sheet opposite,
along the red lines.

Assemble as pictured,
and make yourself a surface of
constant negative curvature!!

Make Yourself A Surface of Constant Negative Curvature!!



This fine product is brought to you by:
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Not responsible for accident or injury.

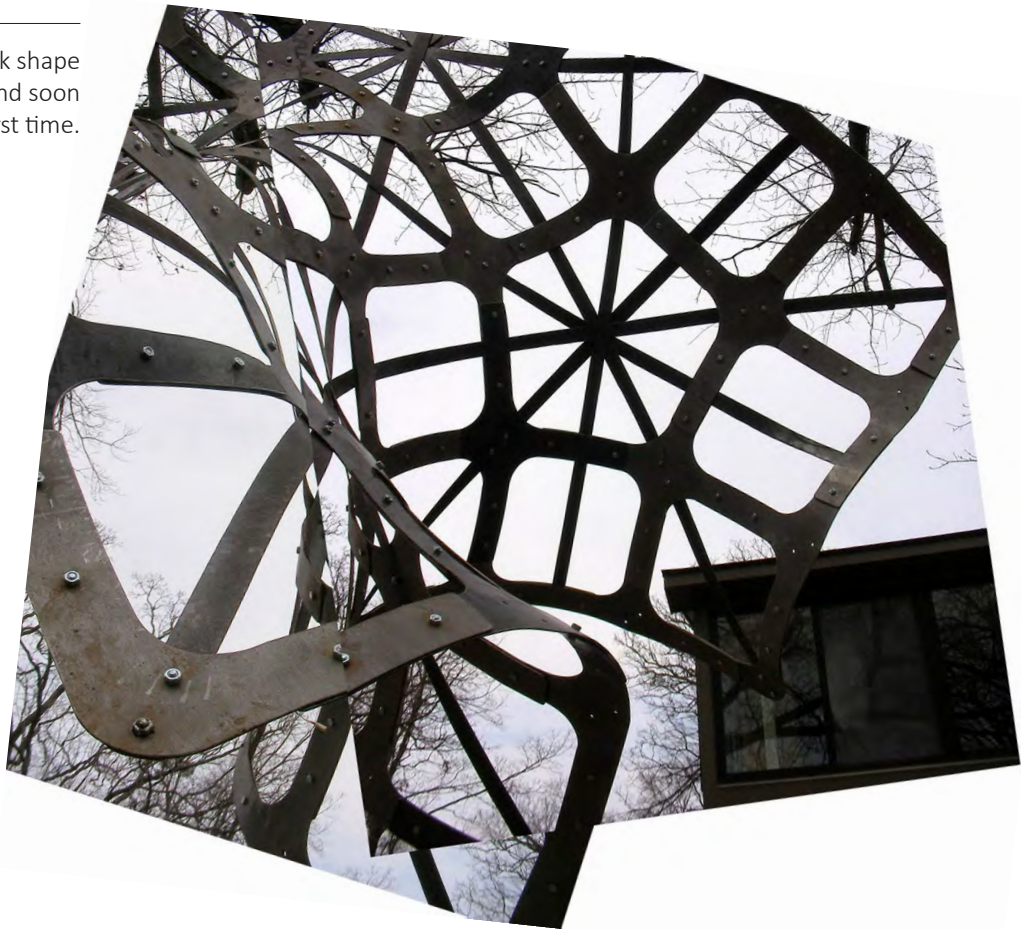
After water-jetting out the pieces, Eugene began to model each one off of a corresponding piece in the paper model.



His specially made bending machine, first used for the Gyring Gyroid, sure came in handy.



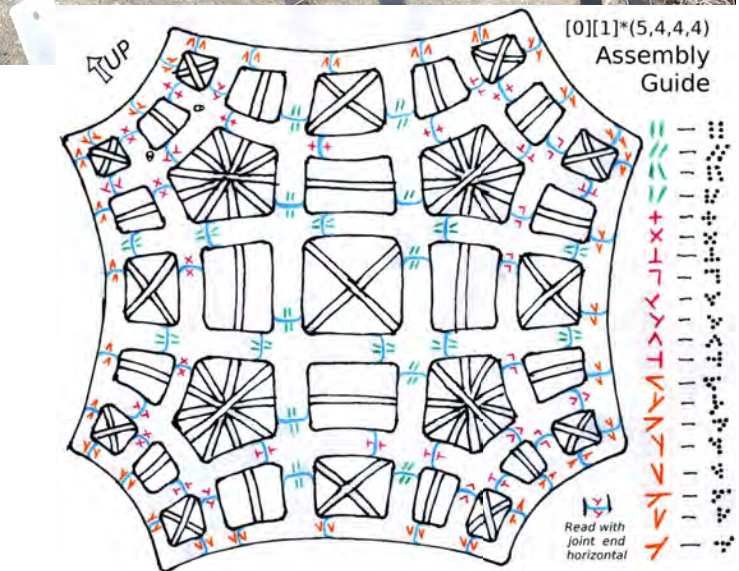
The sculpture took shape in the driveway, and soon was hung for the first time.





... then to be carefully coded,
and taken apart.

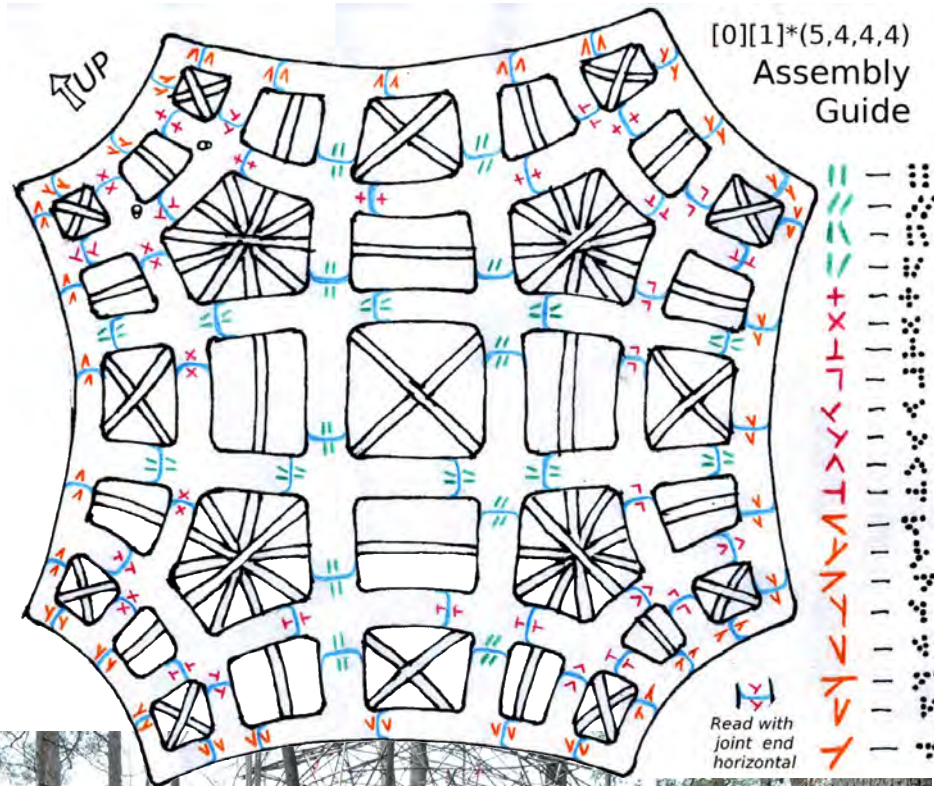
This simple little crib sheet
made everything a snap!



Or maybe not.

This code reads the same upside down as right side up, but not for right-to-left nor up-to-down nor on the diagonal.

It is designed to be easy to drill into dozens of pieces, quickly and to withstand powder coating.



But it made most people cross-eyed, including the sculptors.

Oh dear.



Heroic efforts saved the day!





And soon it came together...











The original paper model (yellow), and the sculpture; the lime green model is at 1/√2 scale to the yellow.





Make your own piece of Hyperbolic Plane!

A SURFACE OF CONSTANT NEGATIVE CURVATURE

by Chaim Goodman-Strauss and Eugene Sargent

Torn Plastic Surface

Daffodil

KALE: It's Hyperbolic!

1 Cut out pieces.

These corners are 85.7 degrees, so when you connect them into a square, something has to give!

Nature creates negative curvature along the edges of leaves and flowers when

2 Connect the ends as shown by arrows with tape or glue.

The paper will warp, creating **NEGATIVE CURVATURE**. The Total Curvature of a disk = $360 - \text{total turning around outside}$.

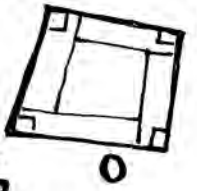
Cells multiply adding extra area. The surface ruffles to accommodate. This can also be observed in the torn edges of plastic trash bags.

Line up the center lines

The material stretches and permanently deforms along the tear, creating a complex pattern that turns out to be a fractal.

3

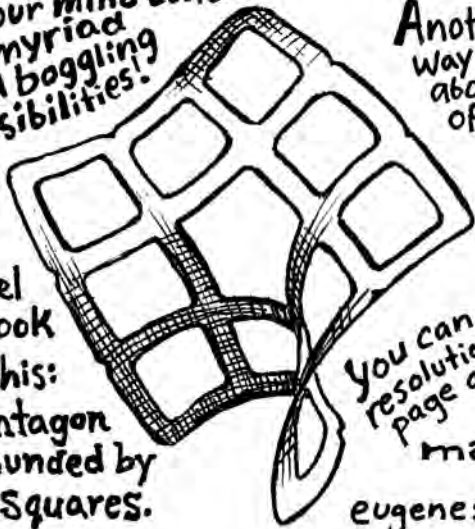
Marvel at the curious thing you have created and let your mind consider the myriad mind boggling possibilities!



As Seen at G4G11 March 2014

this is the last corner of the inside Pentagon

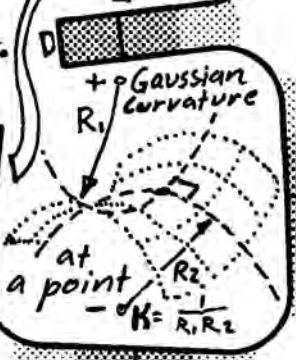
Your Model will look like this: A pentagon surrounded by ten squares.



Another way to think about it: cut strips off the edge of a wavy leaf and flatten them out.



You can get the full resolution file for this page at: mathbun.com and eugenesargent.com/g4g11



The Total Curvature remains constant, and the Gaussian curvature at each point is constant even when the surface flexes.

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